Reteaching
Solving Systems by Graphing

Graphing is useful for solving a system of equations. Graph both equations and look for a point of intersection, which is the solution of that system. If there is no point of intersection, there is no solution.

**Problem**

What is the solution to the system? Solve by graphing. Check.

\[
\begin{align*}
x + y &= 4 \\
2x - y &= 2
\end{align*}
\]

**Solution**

\[
\begin{align*}
y &= -x + 4 \\
y &= 2x - 2
\end{align*}
\]

Put both equations into \( y \)-intercept form, \( y = mx + b \).

The first equation has a \( y \)-intercept of \( (0, 4) \).

Find a second point by substituting in 0 for \( y \) and solve for \( x \).

You have a second point \((4, 0)\), which is the \( x \)-intercept.

The second equation has a \( y \)-intercept of \( (0, -2) \).

Find a second point by substituting in 0 for \( y \) and solve for \( x \).

You have a second point for the second line, \((1, 0)\).

\[
\begin{align*}
x &= 4 \\
y &= 2x - 2 \\
0 &= 2(x) - 2 \\
2 &= 2x, \ x = 1
\end{align*}
\]

Plot both sets of points and draw both lines. The lines appear to intersect \((2, 2)\), so \((2, 2)\) is the solution.

**Check**

If you substitute in the point \((2, 2)\), for \( x \) and \( y \) in your original equations, you can double-check your answer.

\[
\begin{align*}
x + y &= 4 & \quad 2 + 2 &= 4, \quad 4 = 4 \checkmark \\
2x - y &= 2 & \quad 2(2) - 2 &= 2, \quad 2 = 2 \checkmark
\end{align*}
\]
If the equations represent the same line, there is an infinite number of solutions, the coordinates of any of the points on the line.

**Problem**

What is the solution to the system? Solve by graphing. Check.

\[2x - 3y = 6\]
\[4x - 6y = 18\]

**Solution**

What do you notice about these equations? Using the \(y\)-intercepts and solving for the \(x\)-intercepts, graph both lines using both sets of points.

\[y = \frac{2}{3}x - 2\]
\[y = \frac{2}{3}x - 3\]

Graph equation 1 by finding two points: \((0, -2)\) and \((3, 0)\). Graph equation 2 by finding two points \((0, -3)\) and \((4.5, 0)\).

Is there a solution? Do the lines ever intersect? Lines with the same slope are parallel. Therefore, there is no solution to this system of equations.

**Exercises**

Solve each system of equations by graphing. Check.

1. \[2x = 2 - 9y\]
   \[21y = 4 - 6x\]
   \[\left(\frac{1}{2}, \frac{1}{3}\right)\]

2. \[2x = 3 - y\]
   \[y = 4x - 12\]
   \[\left(\frac{3}{2}, -2\right)\]

3. \[y = 1.5x + 4\]
   \[0.5x + y = -2\]
   \[\left(-3, -\frac{1}{2}\right)\]

4. \[6y = 2x - 14\]
   \[x - 7 = 3y\]
   \[\text{infinitely many solutions}\]

5. \[3y = -6x - 3\]
   \[y = 2x - 1\]
   \[\left(0, -1\right)\]

6. \[2x = 3y - 12\]
   \[\frac{1}{3}x = 4y + 5\]
   \[\left(-9, -2\right)\]

7. \[2x + 3y = 11\]
   \[x - y = -7\]
   \[\left(-2, 5\right)\]

8. \[3y = 3x - 6\]
   \[y = x - 2\]
   \[\text{infinitely many solutions}\]

9. \[y = \frac{1}{2}x + 9\]
   \[2y - x = 1\]
   \[\text{no solution}\]