Lesson 3-7: Absolute Value Equations

Name: KEY

In this activity, we will learn to solve **absolute value equations**. An absolute value equation is any equation that contains an absolute value symbol. To start, let's review a little of what we know about the absolute value function.

1. a. $|2| =$ 2 b. $|-2| =$ 2 c. $|-3| =$ 3 d. $|3| =$ 3

2. Circle the numbers in the set S that can be substituted for x to make the equation $|x| = 2$ true.

$$S = \{-3, -2, -1, 0, 1, 2, 3\}$$

3. Circle the numbers in the set S that can be substituted for y to make the equation $|y| = 1$ true.

$$S = \{-3, -2, -1, 0, 1, 2, 3\}$$

Almost every absolute value equation has two answers:
In the equation $|x| = 6$, x could be equal to 6, or x could be equal to -6.
Either value will make the equation true: $|6| = 6$, and $|-6| = 6$

Use the principle above to fill in the blanks for each question below:

4. $|x| = 5$ means that x could be equal to -5 or x could be equal to 5.

5. $|x| = 13$ means that x could be equal to -13 or x could be equal to 13.

6. $|x| = 250$ means that x could be equal to -250 or x could be equal to 250.

Every absolute value equation represents *two* equations combined into one:
$|x| = 10$ means that $x = 10$ or $x = -10$.
$|x - 1| = 7$ means that $x - 1 = 7$ or $x - 1 = -7$

Use this principle to fill in the blanks for each question below:

7. $|x + 3| = 5$ means $x + 3 = 5$ or $x+3=-5$

8. $|2x - 1| = 9$ means $2x-1=9$ or $2x-1=-9$

9. $|5 - 3x| = 25$ means $5-3x=25$ or $5-3x=-25$

10. $\frac{2|5x|}{2} = \frac{10}{2}$ changes to $|5x|=5$ which means $5x=5$ or $5x=-5$

11. $|3e| - 8 = 7$ changes to $|3e|=15$ which means $3e=15$ or $3e=-15$
 $+8 \quad +8$

Skill 3: Solve Absolute Value Equations

To solve absolute value equations, solve the two equations each represents:

$|y-2|=4$ means $y-2=4$ or $y-2=-4$

$$\begin{array}{rcl} y-2=4 & \text{or} & y-2=-4 \\ +2\ \ +2 & & +2\ \ +2 \\ \hline y\ \ =6 & \text{or} & y\ \ =-2 \end{array}$$

The solution set is $y=6$ or $y=-2$

Check: Does $|6-2|=4$? Does $|-2-2|=4$?

$|4|=4$ Yes. $|-4|=4$ Yes.

Use this principle to solve each absolute value equation below and check the solutions.

12. $|x+3|=5$

$x+3=5$ or $x+3=-5$
$-3\ \ -3$ $\quad -3\ \ -3$
$x=2$ $\quad x=-8$
$\boxed{x=2 \text{ or } x=-8}$

13. $|2x-1|=9$

$2x-1=9$ or $2x-1=-9$
$+1\ \ +1$ $\quad +1\ \ +1$
$\frac{2x}{2}=\frac{10}{2}$ $\quad \frac{2x}{2}=\frac{-8}{2}$
$\boxed{x=5 \text{ or } x=-4}$

14. $|5-3x|=25$

$5-3x=25$ or $5-3x=-25$
$-5\ \ -5$ $\quad -5\ \ -5$
$\frac{-3x}{-3}=\frac{20}{-3}$ $\quad \frac{-3x}{-3}=\frac{-30}{-3}$
$\boxed{x=-\frac{20}{3} \text{ or } x=10}$

15. $2|5x|=10$
$\frac{2|5x|}{2}=\frac{10}{2}$
$|5x|=5$

$\frac{5x}{5}=\frac{5}{5}$ or $\frac{5x}{5}=\frac{-5}{5}$
$\boxed{x=1 \text{ or } x=-1}$

16. $|3e|-8=7$
$+8\ \ +8$
$|3e|=15$

$\frac{3e}{3}=\frac{15}{3}$ or $\frac{3e}{3}=\frac{-15}{3}$
$\boxed{e=5 \text{ or } e=-5}$

Scrambled answers for #12-16: $-8, -\frac{20}{3}, -5, -4, -1, 1, 2, 5, 5, 10$

More 3-7: Absolutely Less

Answer each question below:

1. True or False?
   a. $|1| < 2$ b. $|-3| < 2$ c. $|-1| < 2$ d. $|0| < 2$
2. Circle numbers in the set S that can be substituted for x to make the inequality $|x| < 2$ true.
   $S = \{-3, -2.75, -2, -1.33, -1, 0, 0.5, 1, 1.99, 2, 2.66, 3\}$
3. Circle numbers in the set S that can be substituted for y to make the equation $|y| \le 2$ true.
   $S = \{-3, -2.75, -2, -1.33, -1, 0, 0.5, 1, 1.99, 2, 2.66, 3\}$

Every absolute value inequality with < or $\le$ represents *two* inequalities combined with "and":

$|x| < 2$ means that $x < 2$ and $x > -2$.

It means $x < 2$ because for numbers less than 2, the absolute value will be less than 2: $|1| < 2$

It means $x > -2$ because for numbers greater than –2, the absolute value will be less than 2: $|-1| < 2$

The numbers that work in $|x| < 2$ must meet both of these requirements:

-1.7 works because it is less than 2 and greater than –2, so $|-1.7| < 2$ is a true statement.

-3 does not work, even though it is less than 2, because it is not greater than –2, so $|-3| < 2$ is not true.

Use the principle above to fill in the blanks for each question.

4. $|x| \le 5$ means $x \le 5$ and $x \ge -5$
5. $|x| < 250$ means $x < 250$ and $x > -250$
6. $|x+3| < 5$ means $x+3 < 5$ and $x+3 > -5$
7. $|2x| - 1 \le 9$ changes to $|2x| \le 10$ which means $2x \le 10$ and $2x \ge -10$

Skill 4: Solve absolute value inequalities

To solve absolute value inequalities, solve the two inequalities that each represents.

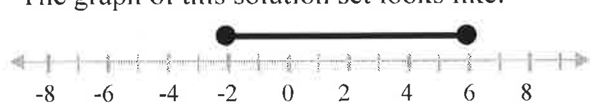
For example, to solve $|y - 2| \le 4$:

$|y - 2| \le 4$ means $y - 2 \le 4$ and $y - 2 \ge -4$

$$\begin{array}{rcl} y - 2 \le 4 & & y - 2 \ge -4 \\ \underline{+2 \quad +2} & & \underline{+2 \quad +2} \\ y \le 6 & \text{and} & y \ge -2 \end{array}$$

The solution set is $y \le 6$ *and* $y \ge -2$, which is the same as $-2 \le y \le 6$.

The graph of this solution set looks like:



Check $x = 5$: Is $|5 - 2| \le 4$?
$|3| \le 4$ Yes.
($x = 5$ is in solution set.)

Check $x = -3$: Is $|-3 - 2| \le 4$?
$|-5| \le 4$ No.
($x = -3$ is not in solution set.)

Use this principle to solve each absolute value inequality, graph the solution set, ~~and check two values – one in the solution set and one not in the solution set.~~

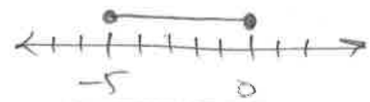
8. $|2x+5| \le 5$

$2x+5 \le 5$ and $2x+5 \ge -5$

$\quad -5 \quad -5 \qquad\qquad -5 \quad -5$

$\frac{2x}{2} \le \frac{0}{2} \qquad \frac{2x}{2} \ge \frac{-10}{2}$

$x \le 0$ and $x \ge -5$



$\boxed{-5 \le x \le 0}$

9. $|4y-8| < 0$

$4y-8<0$ and $4y-8>0$

$\quad +8 \quad +8 \qquad\qquad +8 \quad +8$

$\frac{4y}{4} < \frac{8}{4} \qquad \frac{4y}{4} > \frac{8}{4}$

$y<2$ and $y>2$

$\varnothing$

$\boxed{\text{no solution}}$

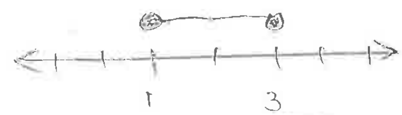
10. $|2-y| \le 1$

$2-y \le 1$ and $2-y \ge -1$

$-2 \quad -2 \qquad\qquad -2 \quad -2$

$\frac{-y}{-1} \le \frac{-1}{-1} \qquad \frac{-y}{-1} \ge \frac{-3}{-1}$

$y \ge 1$ and $y \le 3$



$\boxed{1 \le y \le 3}$

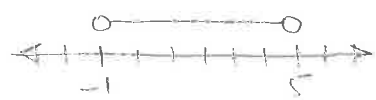
11. $\frac{3|e-2|}{3} < \frac{9}{3}$

$|e-2| < 3$

$e-2<3$ and $e-2>-3$

$\quad +2 \quad +2 \qquad\qquad +2 \quad +2$

$e<5 \qquad e>-1$



$\boxed{-1 < e < 5}$

12. $|3x| + 4 < -8$

$\qquad -4 \quad -4$

$|3x| < -12$

$\frac{3x}{3} < \frac{-12}{3}$ and $\frac{3x}{3} > \frac{12}{3}$

$x < -4$ and $x > 4$

$\varnothing$

$\boxed{\text{no solution}}$

13. $|8-(w-1)| \le 9$

$8-(w-1) \le 9$ and $8-(w-1) \ge -9$

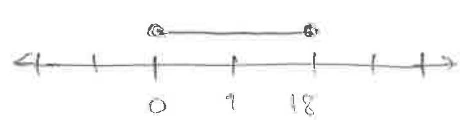
$8-w+1 \le 9 \qquad 8-w+1 \ge -9$

$9-w \le 9 \qquad 9-w \ge -9$

$-9 \quad -9 \qquad\qquad -9 \quad -9$

$\frac{-w}{-1} \le \frac{0}{-1} \qquad \frac{-w}{-1} \ge \frac{-18}{-1}$

$w \ge 0$ and $w \le 18$



$\boxed{0 \le w \le 18}$

Even More 3-7: More Absolutely

Every absolute value inequality with > or ≥ represents *two* inequalities combined with "or":

$|x| \geq 4$ means that $x \geq 4$ or $x \leq -4$.

It means $x \geq 4$ because for numbers greater than 4, the absolute value will be greater than 4: $|6.3| > 4$

It means $x \leq -4$ because for numbers less than –4, the absolute value will be greater than 4: $|-8| > 4$

The numbers that work in $|x| \geq 4$ can meet either of these requirements, but don't have to meet both:

9 works because it is greater than 4, even though it is not less than –4. $|9| \geq 4$ is a true statement.

-7.3 works, even though it is not greater than 4, because it is less than –4. $|-7.3| \geq 4$ is a true statement.

Use the principle above to fill in the blanks for each question below:

1. $|x-1| > 7$ means $x-1>7$ or $x-1<-7$
2. $|5-3x| > 25$ means $5-3x>25$ or $5-3x<-25$
3. $|3e-8| > 7$ means $3e-8>7$ or $3e-8<-7$
4. $|4y| - 8 > 16$ changes to $|4y| > 24$ which means $4y>24$ or $4y<-24$

Skill 4: Solve absolute value inequalities

To solve absolute value inequalities, solve the two inequalities that each represents.

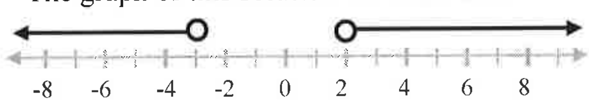
For example, to solve $|2x+1| > 5$:

$|2x+1| > 5$ means

$2x + 1 > 5$	or	$2x + 1 < -5$
$-1 \quad -1$		$-1 \quad -1$
$2x > 4$		$2x < -6$
$x > 2$	or	$x < -3$

The solution set is $x > 2$ *or* $x < -3$.

The graph of this solution set looks like:



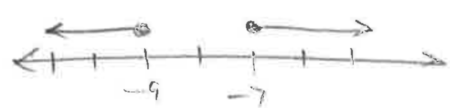
Check $x = -4$: Is $|2(-4)+1| > 5$?
$|-7| > 5$ Yes.
(–4 is in solution set.)

Check $x = 0$: Is $|2(0)+1| > 5$?
$|1| > 5$ No.
(0 is not in solution set.)

Use the principle above to solve each absolute value equation, graph the solution set, ~~and check two values—one in the solution set and one not in the solution set.~~

5. $|w+8| \geq 1$

$w+8 \geq 1$ or $w+8 \leq -1$
$-8 \quad -8$ $\quad -8 \quad -8$
$w \geq -7$ or $w \leq -9$



$\boxed{w \leq -9 \text{ or } w \geq -7}$

6. $|t+4|>3$

$t+4>3$ or $t+4<-3$
$-4 \quad -4$ $\quad$ $-4 \quad -4$
$t>-1$ or $t<-7$

$\boxed{t<-7 \text{ or } t>-1}$

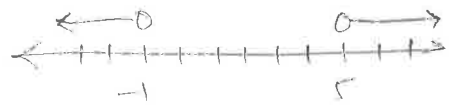
7. $|2y-5|\geq 0$

$2y-5\geq 0$ or $2y-5\leq 0$
$+5 \quad +5$ $\quad$ $+5 \quad +5$
$\frac{2y}{2}\geq\frac{5}{2}$ $\quad$ $\frac{2y}{2}\leq\frac{5}{2}$
$y\geq\frac{5}{2}$ or 2.5 $\quad$ or $\quad$ $y\leq\frac{5}{2}$

$\boxed{\mathbb{R}}$

8. $|6-3x|>9$

$6-3x>9$ or $6-3x<-9$
$-6 \quad -6$ $\quad$ $-6 \quad -6$
$\frac{-3x}{-3}>\frac{3}{-3}$ $\quad$ $\frac{-3x}{-3}<\frac{-15}{-3}$
$x<-1$ or $x>5$



$\boxed{x<-1 \text{ or } x>5}$

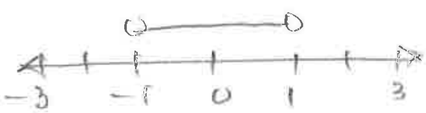
9. $8+|2z|\geq 40$
$-8 \quad -8$
$|2z|\geq 32$

$\frac{2z}{2}\geq\frac{32}{2}$ or $\frac{2z}{2}\leq\frac{-32}{2}$
$z\geq 16$ or $z\leq -16$

$\boxed{z\leq -16 \text{ or } z\geq 16}$

10. $1-3|y|>-2$
$-1 \quad -1$
$\frac{-3|y|}{-3}>\frac{-3}{-3}$
$|y|<1$

$y<1$ and $y>-1$



$\boxed{-1<y<1}$

11. Compare the process of solving inequalities with "<" or "≤" to solving inequalities with ">" or "≥". What is the same? What's different?

The process of solving inequalities is the same whether they have <, ≤, >, or ≥. However, once you get the algebraic statements, you must put them together with the correct word in order to get the correct solution. Once the absolute value is isolated, ≤ and < means use "or", ≥ or > means use "and."