Name ______________________ Class ____________ Date ____________

9-6 Reteaching

The Quadratic Formula and the Discriminant

If a quadratic equation is written in the form $ax^2 + bx + c = 0$, the solutions can be found using the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula is called the **quadratic formula.**

Problem

What are the solutions of $x^2 + 7x = 60$? Use the quadratic formula.

First rewrite the equation in the form $ax^2 + bx + c = 0$.

$x^2 + 7x = 60$

$x^2 + 7x - 60 = 60 - 60$ Subtract 60 from each side.

$x^2 + 7x - 60 = 0$ Simplify.

Therefore, $a = 1$, $b = 7$, and $c = -60$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-60)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{289}}{2}$$

$$x = \frac{-7 \pm 17}{2}$$

The two solutions are $\frac{-7 - 17}{2}$ or -12 and $\frac{-7 + 17}{2}$ or 5.

Exercises

Use the quadratic formula to solve each equation.

1. $x^2 - 19x + 70 = 0$

14; 5

2. $x^2 + 32x + 175 = 0$

-25; -7

3. $2x^2 + 37x - 19 = 0$

-19; 0.5

4. $x^2 - 10x = 75$

15; -5

5. $x^2 + x = 132$

-12; 11

6. $6x^2 + 13x = 28$

-3.5; 1.3

7. $20x^2 + 11x = 3$

$-\frac{3}{4}$, $\frac{1}{5}$

8. $4x^2 + 24x = -35$

-3.5; -2.5

9. $15x^2 + 20 = 40x$

2; $\frac{2}{3}$

9-6 Reteaching (continued)

The Quadratic Formula and the Discriminant

In the quadratic equation, the expression under the radical sign, $b^2 - 4ac$, is called the discriminant. Consider the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2 - 4ac$ is a negative number, the square root cannot be found in the real numbers. There are no real-number solutions of the equation. The graph of the quadratic does not cross the x-axis.
- If $b^2 - 4ac$ equals 0, $x = \frac{-b \pm \sqrt{0}}{2a}$ or $\frac{-b}{2a}$. There is only one solution of the equation. The vertex of the quadratic is on the x-axis.
- If $b^2 - 4ac$ is a positive number, there are two solutions of the equation, $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$. The graph of the quadratic intersects the x-axis twice.

Problem

What is the number of solutions of $x^2 + 13 = -5x$?

First rewrite the equation in the form $ax^2 + bx + c = 0$.

$x^2 + 13 = -5x$

$x^2 + 5x + 13 = 0$ Add $5x$ to each side.

Therefore, $a = 1$, $b = 5$, and $c = 13$.

$$b^2 - 4ac = 5^2 - 4(1)(13)$$
$$= -27$$

Since $b^2 - 4ac$ is a negative number, there are no real-number solutions of the equation.

Exercises

Find the number of solutions of each equation.

10. $4x^2 + 12x + 9 = 0$
one

11. $x^2 - 12x + 32 = 0$
two

12. $x^2 - 10x + 1 = 0$
two

13. $3x^2 + 6x + 8 = 0$
no real solutions

14. $3x^2 - 5x = -6$
no real solutions

15. $x^2 + 100 = 20x$
one

16. $5x^2 - 7x = 2$
two

17. $9x^2 + 4 = 12x$
one

18. $3x^2 + 5x = 2$
two