9-6 Reteaching

The Quadratic Formula and the Discriminant

If a quadratic equation is written in the form $ax^2 + bx + c = 0$, the solutions can be found using the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula is called the **quadratic formula.**

Problem

What are the solutions of $x^2 + 7x = 60$? Use the quadratic formula.

First rewrite the equation in the form $ax^2 + bx + c = 0$.

$x^2 + 7x = 60$

$x^2 + 7x - 60 = 60 - 60$ Subtract 60 from each side.

$x^2 + 7x - 60 = 0$ Simplify.

Therefore, $a = 1$, $b = 7$, and $c = -60$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-60)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{289}}{2}$$

$$x = \frac{-7 \pm 17}{2}$$

The two solutions are $\frac{-7-17}{2}$ or -12 and $\frac{-7+17}{2}$ or 5.

Exercises

Use the quadratic formula to solve each equation.

1. $x^2 - 19x + 70 = 0$

2. $x^2 + 32x + 175 = 0$

3. $2x^2 + 37x - 19 = 0$

4. $x^2 - 10x = 75$

5. $x^2 + x = 132$

6. $6x^2 + 13x = 28$

7. $20x^2 + 11x = 3$

8. $4x^2 + 24x = -35$

9. $15x^2 + 20 = 40x$

In the quadratic equation, the expression under the radical sign, $b^2 - 4ac$, is called the discriminant. Consider the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2 - 4ac$ is a negative number, the square root cannot be found in the real numbers. There are no real-number solutions of the equation. The graph of the quadratic does not cross the x-axis.
- If $b^2 - 4ac$ equals 0, $x = \frac{-b \pm \sqrt{0}}{2a}$ or $\frac{-b}{2a}$ There is only one solution of the equation. The vertex of the quadratic is on the x-axis.
- If $b^2 - 4ac$ is a positive number, there are two solutions of the equation, $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ The graph of the quadratic intersects the x-axis twice.

Problem

What is the number of solutions of $x^2 + 13 = -5x$?

First rewrite the equation in the form $ax^2 + bx + c = 0$.

$x^2 + 13 = -5x$

$x^2 + 5x + 13 = 0$ Add 5x to each side.

Therefore, $a = 1$, $b = 5$, and $c = 13$.

$b^2 - 4ac = 5^2 - 4(1)(13)$

$= -27$

Since $b^2 - 4ac$ is a negative number, there are no real-number solutions of the equation.

Exercises

Find the number of solutions of each equation.

10. $4x^2 + 12x + 9 = 0$

11. $x^2 - 12x + 32 = 0$

12. $x^2 - 10x + 1 = 0$

13. $3x^2 + 6x + 8 = 0$

14. $3x^2 - 5x = -6$

15. $x^2 + 100 = 20x$

16. $5x^2 - 7x = 2$

17. $9x^2 + 4 = 12x$

18. $3x^2 + 5x = 2$