

9-5 Reteaching

Completing the Square

You have learned to square binomials. Notice how the coefficient of the a term is related to the constant value in every perfect-square trinomial.

$$(a + 1)^2 = (a + 1)(a + 1) = a^2 + 2a + 1 \rightarrow \left(\frac{2}{2}\right)^2 = 1$$

$$(a - 1)^2 = (a - 1)(a - 1) = a^2 - 2a + 1 \rightarrow \left(\frac{-2}{2}\right)^2 = 1$$

$$(a - 2)^2 = (a - 2)(a - 2) = a^2 - 4a + 4 \rightarrow \left(\frac{-4}{2}\right)^2 = 4$$

$$(a + 3)^2 = (a + 3)(a + 3) = a^2 + 6a + 9 \rightarrow \left(\frac{6}{2}\right)^2 = 9$$

In each case, half the coefficient of the a term squared equals the constant term. You can use this pattern to find the value that makes a trinomial a perfect square.

Problem

What is the value of c such that $x^2 - 14x + c$ is a perfect-square trinomial?

The coefficient of the x term is -14 . Using the pattern, $c = \left(\frac{-14}{2}\right)^2$ or 49 .

So, $x^2 - 14x + 49$ is a perfect-square trinomial.

Exercises

Find the value of c such that each expression is a perfect-square trinomial.

1. $a^2 + 8a + c$

2. $x^2 - 16x + c$

3. $m^2 + 20m + c$

4. $p^2 - 14p + c$

5. $y^2 - 10y + c$

6. $b^2 + 18b + c$

7. $d^2 + 12d + c$

8. $n^2 - n + c$

9. $w^2 + 3w + c$

You can use completing the square to solve quadratic equations.

Problem

What are the solutions of the equation $x^2 + 2x - 48 = 0$?

First rewrite the equation so that the constant is on one side of the equation and the other terms are on the other side.

$$x^2 + 2x - 48 = 0$$

$$x^2 + 2x - 48 + 48 = 0 + 48$$

Add 48 to each side.

$$x^2 + 2x = 48$$

Simplify.

Since $\left(\frac{2}{2}\right)^2 = 1$, add 1 to each side.

$$x^2 + 2x + 1 = 48 + 1$$

Add 1 to each side.

$$(x + 1)^2 = 49$$

Simplify.

$$x + 1 = \pm\sqrt{49}$$

Take the square root of each side.

$$x + 1 = \pm 7$$

Simplify.

$$x + 1 = -7$$

or

$$x + 1 = 7$$

$$x + 1 - 1 = -7 - 1$$

or

$$x + 1 - 1 = 7 - 1$$

$$x = -8$$

or

$$x = 6$$

The solutions are -8 and 6 .

Exercises

Solve each equation by completing the square. If necessary, round to the nearest hundredth.

10. $b^2 + 10b = 75$

11. $y^2 - 18y = 63$

12. $n^2 - 20n = -75$

13. $a^2 + 16a = -15$

14. $t^2 + 8t - 9 = 0$

15. $h^2 - 12h - 9 = 0$

16. $m^2 - 2m - 8 = 0$

17. $s^2 + 6s + 1 = 0$

18. $v^2 + 4v - 2 = 0$