

8-8

Reteaching

Factoring by Grouping

You can factor some higher-degree polynomials by grouping terms and factoring out the GCF to find the common binomial factor. Make sure to factor out a common GCF from all terms first before grouping.

Problem

What is the factored form of $2b^4 - 8b^3 + 10b^2 - 40b$?

$$\begin{aligned}
 2b^4 - 8b^3 + 10b^2 - 40b &= 2b(b^3 - 4b^2 + 5b - 20) && 2b \text{ is the GCF of all four terms. Factor out } 2b \\
 &&& \text{from each term.} \\
 &= 2b[b^2(b - 4) + 5(b - 4)] && \text{Group terms into pairs and look for the GCF of} \\
 &&& \text{each pair. } b^2 \text{ is the GCF of the first pair, and } 5 \\
 &&& \text{is the GCF of the second pair.} \\
 &= 2b(b^2 + 5)(b - 4) && b - 4 \text{ is the common binomial factor. Use the} \\
 &&& \text{Distributive Property to rewrite the expression.}
 \end{aligned}$$

Multiply to check your answer.

$$\begin{aligned}
 2b(b^2 + 5)(b - 4) &= 2b(b^3 + 5b - 4b^2 - 20) && \text{Multiply } b^2 + 5 \text{ and } b - 4. \\
 &= 2b^4 + 10b^2 - 8b^3 - 40b && \text{Multiply by } 2b. \\
 &= 2b^4 - 8b^3 + 10b^2 - 40b \checkmark && \text{Reorder the terms by degree.}
 \end{aligned}$$

The factored form of $2b^4 - 8b^3 + 10b^2 - 40b$ is $2b(b^2 + 5)(b - 4)$.

Exercises

Factor completely. Show your steps.

- $4x^4 + 8x^3 + 12x^2 + 24x$
 $4x(x^2 + 3)(x + 2)$
- $24y^4 + 6y^3 + 36y^2 + 9y$
 $3y(2y^2 + 3)(4y + 1)$
- $72z^4 + 48z^3 + 126z^2 + 84z$
 $6z(4z^2 + 7)(3z + 2)$
- $2e^4 - 8e^3 + 18e^2 - 72e$
 $2e(e^2 + 9)(e - 4)$
- $12f^3 - 36f^2 + 60f - 180$
 $12(f^2 + 5)(f - 3)$
- $16g^4 - 56g^3 + 64g^2 - 224g$
 $8g(g^2 + 4)(2g - 7)$
- $56m^3 - 28m^2 - 42m + 21$
 $7(4m^2 - 3)(2m - 1)$
- $40n^4 - 60n^3 - 50n^2 + 75n$
 $5n(4n^2 - 5)(2n - 3)$
- $60x^3 - 90x^2 - 30x + 45$
 $15(2x^2 - 1)(2x - 3)$
- $12p^5 + 8p^4 + 18p^3 + 12p^2$
 $2p^2(2p^2 + 3)(3p + 2)$
- $6r^3 + 9r^2 - 60r$
 $3r(2r - 5)(r + 4)$
- $20s^6 - 50s^5 - 30s^4$
 $10s^4(2s + 1)(s - 3)$

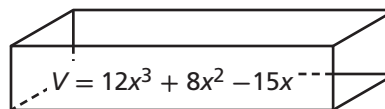
8-8 **Reteaching** (continued)

Factoring by Grouping

Polynomials can be used to express the volume of a rectangular prism. They can sometimes be factored into 3 expressions to represent possible dimensions of the prism. The three factors are the length, width, and height.

Problem

The plastic storage container to the right has a volume of $12x^3 + 8x^2 - 15x$. What linear expressions could represent possible dimensions of the storage container?



$$\begin{aligned}
 12x^3 + 8x^2 - 15x &= x(12x^2 + 8x - 15) \\
 &= x(12x^2 + 18x - 10x - 15) \\
 &= x[6x(2x + 3) - 5(2x + 3)] \\
 &= x(6x - 5)(2x + 3)
 \end{aligned}$$

Factor out x , the GCF for all three terms.
 ac is -180 and b is 8 . Break $8x$ into two terms that have a sum of $8x$ and a product of $-180x^2$.
 Group the terms into pairs and factor out the GCF from each pair. The GCF of the first pair is $6x$. The GCF of the second pair is -5 .
 $2x + 3$ is the common binomial term. Use the Distributive Property to reorganize the factors.

Multiply to check your answer.

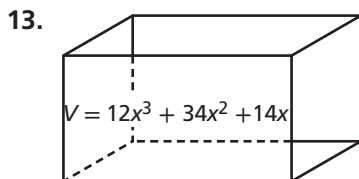
$$\begin{aligned}
 x(6x - 5)(2x + 3) &= x(12x^2 + 18x - 10x - 15) \\
 &= x(12x^2 + 8x - 15) \\
 &= 12x^3 + 8x^2 - 15x \checkmark
 \end{aligned}$$

Multiply $6x - 5$ and $2x + 3$.
 Combine like terms.
 Multiply by x .

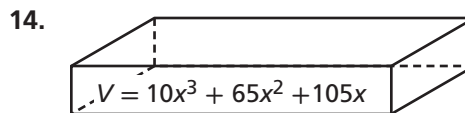
Possible dimensions of the storage container are x , $6x - 5$, and $2x + 3$.

Exercises

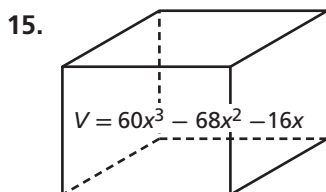
Find linear expressions for the possible dimensions of each rectangular prism.



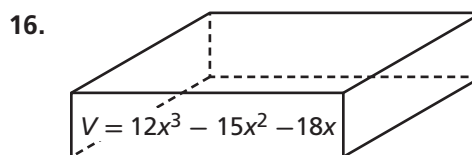
2x, 3x + 7, 2x + 1



5x, 2x + 7, x + 3



4x, 5x + 1, 3x - 4



3x, 4x + 3, x - 2