

8-6 Reteaching

Factoring $ax^2 + bx + c$

You can use your knowledge of prime numbers to help you factor some trinomials as two binomials. A prime number has only 1 and itself as factors. For trinomials of the form $ax^2 + bx + c$, if a is a prime number then you already know the first term of each binomial: ax and $1x$. Then list the factors that will multiply to produce c . Use guess and check to find the factor pair that will add to b .

Problem

What is the factored form of $7x^2 + 31x + 12$?

$$\begin{aligned} 7x^2 + 31x + 12 &= (7x \quad)(1x \quad) && a \text{ is } 7, \text{ which is prime, so the factors are } 7 \text{ and } 1. \\ &= (7x \quad)(x \quad) && \text{You don't need the } 1 \text{ in front of the variable, so drop it.} \end{aligned}$$

$$7x^2 + 31x + 12 = (7x + \quad)(x + \quad) \quad \text{The trinomial has two plus signs, so the binomials also have plus signs.}$$

Because c is 12, find factor pairs that multiply to 12: (1 and 12), (2 and 6), (3 and 4).

Try each pair in the expression to see if the INNER and OUTER products add to b , or 31.

$$(7x + 1)(x + 12) = 7x^2 + x + 84x = 7x^2 + \mathbf{85}x + 12 \quad (\text{NO})$$

$$(7x + 2)(x + 6) = 7x^2 + 2x + 42x = 7x^2 + \mathbf{44}x + 12 \quad (\text{NO})$$

$$(7x + 3)(x + 4) = 7x^2 + 3x + 28x = 7x^2 + \mathbf{31}x + 12 \quad (\text{YES})$$

The factored form of $7x^2 + 31x + 12$ is $(7x + 3)(x + 4)$.

Exercises

Factor each expression.

1. $3x^2 + 14x + 8$

$(3x + 2)(x + 4)$

2. $5y^2 + 43y + 24$

$(5y + 3)(y + 8)$

3. $2z^2 + 19z + 42$

$(2z + 7)(z + 6)$

4. $11a^2 + 39a + 18$

$(11a + 6)(a + 3)$

5. $13b^2 + 58b + 24$

$(13b + 6)(b + 4)$

6. $23c^2 + 56c + 20$

$(23c + 10)(c + 2)$

7. $7d^2 + d - 8$

$(7d + 8)(d - 1)$

8. $3e^2 + 20e - 32$

$(3e - 4)(e + 8)$

9. $19f^2 + 10f - 9$

$(19f - 9)(f + 1)$

10. $5s^2 - 18s + 16$

$(5s - 8)(s - 2)$

11. $17t^2 - 12t - 5$

$(17t + 5)(t - 1)$

12. $29u^2 + 48u - 20$

$(29u - 10)(u + 2)$

8-6

Reteaching (continued)

Factoring $ax^2 + bx + c$

If you are given the area and one side of a rectangle, you can find the second side by factoring the trinomial. One binomial is the width and the other binomial is the length.

Problem

The area of a rectangular swimming pool is $6x^2 + 11x + 3$. The width of the pool is $2x + 3$. What is the length of the pool?

You are given the area and length of the pool. Set up an equation with what you are given and solve for length.

$$6x^2 + 11x + 3 = (2x + 3)(\square\square\square)$$

Area = length \times width.

$$6x^2 + 11x + 3 = (2x + 3)(3x\square\square)$$

$6x^2 = (2x)(3x)$, so the first term of the second binomial is $3x$.

$$6x^2 + 11x + 3 = (2x + 3)(3x + \square)$$

The trinomial has two plus signs, so the sign for the second binomial must also be plus.

$$6x^2 + 11x + 3 = (2x + 3)(3x + 1)$$

The value of c is 3. Since $3 = 3 \times 1$, the second term must be 1.

Multiply to check your answer. Use FOIL.

$$(2x + 3)(3x + 1) = 6x^2 + 2x + 9x + 3 = 6x^2 + 11x + 3 \checkmark$$

The length of the swimming pool is $3x + 1$.

Exercises

13. The area of a rectangular cookie sheet is $8x^2 + 26x + 15$. The width of the cookie sheet is $2x + 5$. What is the length of the cookie sheet?

$$4x + 3$$

14. The area of a rectangular lobby floor in the new office building is $15x^2 + 47x + 28$. The length of one side of the lobby is $5x + 4$. What is the width?

$$3x + 7$$

15. The area of a rectangular school banner is $12x^2 + 13x - 90$. The width of the banner is $3x + 10$. What is the length of the banner?

$$4x - 9$$

16. The distance a train has traveled is $6x^2 - 23x + 20$. The train's average speed is $3x - 4$. How long has the train been traveling?

$$2x - 5$$