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Reteaching

Rational Exponents and Radicals

When you understand the equivalence of radicals and rational exponents, you can represent a radical expression using rational exponents.

If the n th root of a is a real number and m and n are positive integers, then

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

You can also represent an expression with rational exponents as a radical expression. Sometimes converting from one form to the other makes it easier to simplify an expression involving roots.

Problem

What is $8^{\frac{2}{3}}$ in radical form? Simplify, if possible.

$$8^{\frac{2}{3}} = \sqrt[3]{8^2}$$

Rewrite the expression in radical form.

$$= \sqrt[3]{8^2} = \sqrt[3]{64} = \sqrt[3]{4 \cdot 4 \cdot 4} = 4$$

Simplify.

Write each expression in radical form, then simplify if possible.

1. $36^{\frac{1}{2}}$
 $\sqrt{36}; 6$

2. $4^{\frac{5}{2}}$
 $\sqrt{4^5}; 32$

3. $(27x)^{\frac{2}{3}}$
 $\sqrt[3]{(27x)^2}; 9\sqrt[3]{x^2}$

4. $18b^{\frac{5}{3}}$
 $18\sqrt[3]{b^5}$

Problem

What is $\sqrt[4]{(16x)^3}$ in exponential form? Simplify, if possible.

$$\sqrt[4]{(16x)^3} = (16x)^{\frac{3}{4}}$$

Rewrite the expression in exponential form.

$$= 16^{\frac{3}{4}}x^{\frac{3}{4}} = 8x^{\frac{3}{4}}$$

Simplify.

Write each expression in exponential form, then simplify if possible.

5. $\sqrt[3]{8^2}$
 $8^{\frac{2}{3}}; 4$

6. $\sqrt[3]{8x}$
 $(8x)^{\frac{1}{3}}; 2x^{\frac{1}{3}}$

7. $\sqrt[3]{64x^7}$
 $64^{\frac{1}{3}}x^{\frac{7}{3}}; 4x^{\frac{7}{3}}$

8. $\sqrt[3]{(27x)^2}$
 $(27x)^{\frac{2}{3}}; 9x^{\frac{2}{3}}$

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Reteaching (continued)

Rational Exponents and Radicals

You can use the properties of exponents to simplify expressions with rational exponents. When simplifying radical expressions, you can combine like terms.

Problem

What is the simplified form of the expression, $\left(a^{\frac{1}{4}}\right)\left(a^{\frac{1}{4}}\right)$? Use the properties of exponents, and then write the expression in radical form.

$$\begin{aligned}\left(a^{\frac{1}{4}}\right)\left(a^{\frac{1}{4}}\right) &= a^{\frac{1}{4}+\frac{1}{4}} = a^{\frac{1}{2}} \\ &= \sqrt{a}\end{aligned}$$

When multiplying like bases, add the exponents.

Rewrite in radical form.

Simplify each expression using the properties of exponents, and then write the expression in radical form.

9. $\left(a^{\frac{2}{3}}\right)\left(a^{\frac{1}{4}}\right)$ $a^{\frac{11}{12}}$; $\sqrt[12]{a^{11}}$

10. $(ab)^{\frac{1}{3}}\left(b^{\frac{1}{2}}\right)$ $a^{\frac{1}{3}}b^{\frac{5}{6}}$; $\sqrt[3]{a}\sqrt[6]{b^5}$

11. $\left(5x^{\frac{1}{5}}\right)(x^2)$ $5x^{\frac{11}{5}}$; $5\sqrt[5]{x^{11}}$

12. $(8x)^{\frac{1}{3}}(64x)^{\frac{1}{2}}$ $16x^{\frac{5}{6}}$; $16\sqrt[6]{x^5}$

Problem

What is the exponential form of the expression, $\sqrt{a^3} + 3\sqrt{a^5}$?

$$\sqrt{a^3} + 3\sqrt{a^5} = a^{\frac{3}{2}} + 3a^{\frac{5}{2}}$$

Rewrite the expression in exponential form. The expression cannot be simplified further because the terms are not like terms.

Write each expression in exponential form. Simplify when possible.

13. $\sqrt[3]{a^2} + 2\sqrt[3]{a^2}$ $3a^{\frac{2}{3}}$

14. $\left(\sqrt{16x^4}\right) + \left(\sqrt[3]{27x^5}\right)$ $4x^2 + 3x^{\frac{5}{3}}$

15. $\sqrt[3]{(27x)^2} + \sqrt[4]{256x^2}$ $9x^{\frac{2}{3}} + 4x^{\frac{1}{2}}$

16. $4\sqrt[3]{(3x)^6} - \sqrt{(2x)^4}$ $32x^2$