Lesson 9-5 Completing the Square

Name ________________________

In the next two lessons, you will be solving quadratic equations by a couple of new methods that often produce solutions involving square roots. In Lesson 9-5A, you will learn a method for putting radicals into their simplest form, called **simplifying radicals**.

List the first 10 perfect squares: ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___

Let's review a little of what we know about radicals, or square roots. Find each root below:

1. $\sqrt{4}$ =
2. $\sqrt{25}$ =
3. $\sqrt{100}$ =
4. $\sqrt{16}$ =
5. $\sqrt{9}$ =
6. $\sqrt{144}$ =
7. $\sqrt{36}$ =
8. $\sqrt{49}$ =
9. $\sqrt{1764}$ =

Use your answers to the questions above to answer each question below:

10. Does $\sqrt{4}\cdot\sqrt{25}=\sqrt{4\cdot 25}$? Explain how you know.

11. Does $\sqrt{16}\cdot\sqrt{9}=\sqrt{16\cdot 9}$? Explain how you know.

12. Does $\sqrt{36}\cdot\sqrt{49}=\sqrt{36\cdot 49}$? Explain how you know.

These questions call attention to an important principle about how square roots work.

Product of Roots Rule

The product of two roots is equal to the root of the product.

$$\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}$$

Use the principle above to find the answer to these questions involving roots:

13. $\sqrt{4}\cdot\sqrt{3}=\sqrt{\quad}$
14. $\sqrt{9}\cdot\sqrt{2}=\sqrt{\quad}$
15. $\sqrt{25}\cdot\sqrt{3}=\sqrt{\quad}$
16. $\sqrt{36}\cdot\sqrt{7}=\sqrt{\quad}$
17. $\sqrt{49}\cdot\sqrt{6}=\sqrt{\quad}$
18. $\sqrt{144}\cdot\sqrt{5}=\sqrt{\quad}$

This principle is most useful when we apply it in reverse. Instead of multiplying smaller roots to get one that is even larger and more difficult, we should try taking a large root and break it down into a product of smaller roots that we can do separately.

$\sqrt{12}=\sqrt{4\cdot 3}=\sqrt{4}\cdot\sqrt{3}=2\cdot\sqrt{3}$

So $\sqrt{12}=2\sqrt{3}$

$\sqrt{18}=\sqrt{9\cdot 2}=\sqrt{9}\cdot\sqrt{2}=3\cdot\sqrt{2}$

So $\sqrt{18}=3\sqrt{2}$

We say that these answers are in **simplest radical form**. In other words, even though the numbers we started with were not perfect squares, we were able to find a perfect square that divided into them, and we did the square root of that number.
The following box summarizes how this method for simplifying radicals works:

Simplifying Radicals by Perfect Squares

Step 1: Find the largest perfect square that divides into the number under the radical.

Step 2: Write the number under the radical as a product of this perfect square and another number.

Step 3: Write this radical of a product as the product of the two radicals.

Step 4: Do the square root of the perfect square and multiply this number by the remaining radical.

<u>Example</u>: Simplify $\sqrt{75}$

Step 1: 25 is the largest perfect square that divides into 75. (It goes in 3 times.)

Step 2: $\sqrt{75} = \sqrt{25 \cdot 3}$

Step 3: $\sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3}$

Step 4: $\sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$ Therefore, $\sqrt{75} = 5\sqrt{3}$

Use the process described above to simplify each radical below:

19. $\sqrt{50}$

20. $\sqrt{98}$

21. $\sqrt{8}$

22. $\sqrt{32}$

23. $\sqrt{72}$

24. $\sqrt{45}$

25. $\sqrt{200}$

26. $\sqrt{192}$

27. $\sqrt{162}$

28. $\sqrt{288}$

29. $\sqrt{847}$

30. $\sqrt{845}$

More Lesson 9-5 Perfect Squares

Whenever you multiply two of the same item, you "square" it. The result is called a **perfect square**. Since $4 = 2 \cdot 2 = 2^2$, 4 is a *perfect square*. Since $9x^2 = 3x \cdot 3x = (3x)^2$, $9x^2$ is a *perfect square*. Sometimes factoring polynomials can give us perfect squares.

For example, since $x^2 + 6x + 9 = (x+3)(x+3) = (x+3)^2$, $x^2 + 6x + 9$ is also a *perfect square*. Since it has three terms, it is called a **perfect square trinomial**.

Factor each of the following *perfect square trinomials*:

1. $x^2 + 12x + 36 = (x + \_\_\_)(x + \_\_\_) = (x + \_\_\_)^2$

2. $y^2 - 20y + 100 = (y - \_\_\_\ )(y - \_\_\_\ ) =$

3. $z^2 + 8z + 16 =$

4. $a^2 - 14a + 49 =$

Fill in the blanks to make each trinomial a *perfect square*:

5. $x^2 + \_\_\_\_ + 25 = (x + \_\_\_)^2$

6. $b^2 + \_\_\_\_ + 16 = (\_\_\_\_\_\_\_\_)^2$

7. $c^2 - \_\_\_\_ + 49 = (\_\_\_\_\_\_\_\_)^2$

8. $d^2 - \_\_\_\_ + 144 = (\_\_\_\_\_\_\_\_)^2$

9. How is the middle number of the trinomial related to the last number of the trinomial?

Fill in the blanks to make each trinomial a *perfect square*:

10. $x^2 + 18x + \_\_\_\_ = (x + \_\_\_)^2$

11. $e^2 + 6e + \_\_\_\_ = (\_\_\_\_\_\_\_\_)^2$

12. $f^2 - 16f + \_\_\_\_ = (\_\_\_\_\_\_\_\_)^2$

13. $g^2 - 12g + \_\_\_\_ = (\_\_\_\_\_\_\_\_)^2$

14. How is the last number of the trinomial related to the middle number of the trinomial?

The process of finding the missing value to make a *perfect square trinomial*, like we did in #10-13, is called **completing the square**. In the next activity, you will learn how we can use this idea to help us solve quadratic equations.

More Lesson 9-5 Completing the Square

The process of turning a polynomial into a *perfect square* is called **completing the square**. This idea can be used to help us solve quadratic equations, even ones that don't factor. It works because when things are squared, we can undo the square by "unsquaring", like we did in a previous activity.

Solving quadratic equations by completing the square

Step 1: Move the constant, if any, to the other side of the equation from the squared term.
Step 2: Find half the coefficient of the linear term, square it, and add this value to both sides.
Step 3: Complete the square.
Step 4: "Unsquare" by taking the square root of both sides.
Step 5: Solve for the variable.

Example 1: Solve $x^2+2x-7=0$

Step 1: $x^2+2x-7=0$

$\quad +7 \quad +7$

$x^2+2x \quad =7$

Step 2: $\frac{2}{2}=1$, $1^2=1$, so we get $x^2+2x \quad =7$

$\quad +1 \quad +1$

$x^2+2x+1=8$

Step 3: $x^2+2x+1=8$

$(x+1)^2=8$

Step 4: $\sqrt{(x+1)^2}=\pm\sqrt{8}$

Step 5: $x+1=\pm\sqrt{2\cdot 2\cdot 2}$

$\quad -1 \quad -1$

$x=-1\pm 2\sqrt{2}$

Example 2: Solve $j^2-14j-6=5$

Step 1: $j^2-14j-6=5$

$\quad +6 \quad +6$

$j^2-14j \quad =11$

Step 2: $j^2-14j \quad =11$

$\quad +49 \quad +49$

$j^2-14j+49=60$

Step 3: $j^2-14j+49=60$

$(j-7)^2=60$

Step 4: $\sqrt{(j-7)^2}=\pm\sqrt{60}$

Step 5: $j-7=\pm\sqrt{2\cdot 2\cdot 3\cdot 5}$

$\quad +7 \quad +7$

$j=7\pm 2\sqrt{15}$

Use the process described above to solve each quadratic equation. Be sure to check your answers with the scrambled answers at the bottom of the next page.

1. $x^2+4x-3=0$

2. $a^2+10a=10$

3. $b^2 - 6b + 1 = 0$

4. $c^2 - 12c = -4$

5. $d^2 + 2d - 7 = 1$

6. $e^2 + 8e - 2 = -10$

7. $f^2 + 20f + 50 = 200$

8. $g^2 + 18g - 40 = 4$

9. $2h^2 + 20h + 50 = 200$

10. $3k^2 - 36k - 24 = 300$

Scrambled answers for 1-10:

$\{-15,\ 5\}$, $\{-9 \pm 5\sqrt{5}\}$, $\{-10 \pm 5\sqrt{10}\}$, $\{-6,\ 18\}$, $\{-5 \pm \sqrt{35}\}$, $\{-4 \pm 2\sqrt{2}\}$, $\{-4,\ 2\}$, $\{-2 \pm \sqrt{7}\}$, $\{3 \pm 2\sqrt{2}\}$, $\{6 \pm 4\sqrt{2}\}$