Lesson 7-5A The Root of the Problem

How would exponents work if they were fractions? To find out, we we will use the internet concept, where the base number represents what the amount of information gets multiplied by every year. Since the exponent represents time, we will be looking at situations where less than a full year has passed, so the exponent is a fraction.

1. Suppose the amount of information available on the web is multiplied by 4 every year. How much information will be available …

   a. $\frac{1}{2}$ year from now? $\underline{4^{\frac{1}{2}}}$

   b. after two half-years have passed? $\underline{4^{\frac{1}{2}}} \times \underline{4^{\frac{1}{2}}} = 4^1$ $(b^m \cdot b^n = b^{m+n})$

   c. Find the missing value. ______ × ______ = 4

   d. Therefore, $4^{\frac{1}{2}} =$ ______

2. Suppose the amount of information available on the web is multiplied by 9 every year. How much information will be available …

   a. $\frac{1}{2}$ year from now? _____

   b. after two half-years have passed? ____ × ____ = 9^1

   c. Find the missing value. ______ × ______ = 9

   d. Therefore, $9^{\frac{1}{2}} =$ ______

3. Supppose the amount of information available on the web is multiplied by 27 every year. How much information will be available …

   a. $\frac{1}{3}$ year from now? ____

   b. after three $^1/_3$ years have passed? _____ × ____ × ____ = 27^1

   c. Find the missing value. ______ × ______ × ______ = 27

   d. Therefore, $27^{\frac{1}{3}} =$ ______

4. Supppose the amount of information available on the web is multiplied by 16 every year. How much information will be available …

   a. $\frac{1}{4}$ year from now? ____

   b. after four $^1/_4$ years have passed? ____ × ____ × ____ × ____ = 16^1

   c. Find the missing value. ______ × ______ × ______ × ______ = 16

   d. Therefore, $16^{\frac{1}{4}} =$ ______

5. Use the same kind of reasoning to estimate each of the following:

a. $16^{\frac{1}{2}} =$
b. $49^{\frac{1}{2}} =$

c. $36^{\frac{1}{2}} =$
d. $81^{\frac{1}{2}} =$

e. $8^{\frac{1}{3}} =$
f. $64^{\frac{1}{3}} =$

g. $216^{\frac{1}{3}} =$
h. $81^{\frac{1}{4}} =$

6. The answers in question #4 should seem rather familiar! Think about these:

a. $\sqrt{16} =$____ because $4 \cdot 4 =$ ____
b. $\sqrt{49} =$_____ because $7 \cdot 7 =$ ____

c. $\sqrt{36} =$_____ because ______ = 36
d. $\sqrt{81} =$_____ because ______ = ___

e. $\sqrt[3]{8} =$____ because $2 \cdot 2 \cdot 2$ = ____ ______
f. $\sqrt[3]{64} =$______ because ___ _______ = ____

g. $\sqrt[3]{216} =$___ because ________ = ____
h. $\sqrt[4]{81} =$ ______ because _________ = ____

Now you can see that there is a very strong connection between fractional exponents and roots!

Fractional Exponents Rule #1

The denominator of a fraction exponent indicates a root:

$b^{\frac{1}{2}} = \sqrt{b}$, $b^{\frac{1}{3}} = \sqrt[3]{b}$, and in general, $b^{\frac{1}{n}} = \sqrt[n]{b}$

7. Use this exponent rule to evaluate each of the following:

a. $25^{\frac{1}{2}} = \sqrt{\quad} =$
b. $100^{\frac{1}{2}} = \sqrt{\quad} =$

c. $100^{\frac{1}{2}} = \sqrt{\quad} =$
d. $125^{\frac{1}{3}} = \sqrt[3]{\quad} =$

e. $216^{\frac{1}{3}} = \sqrt[3]{\quad} =$
f. $81^{\frac{1}{4}} = \sqrt[4]{\quad} =$

g. $256^{\frac{1}{4}} = \sqrt[4]{\quad} =$
h. $32^{\frac{1}{5}} = \sqrt[5]{\quad} =$

8. Use this rule in reverse to fill in the blank for each problem:

a. $8 = \sqrt{\quad}$
b. $11 = \sqrt{\quad}$

c. $15 = \sqrt{\quad}$
d. $20 = \sqrt{\quad}$

e. $5 = \sqrt[3]{\quad}$
f. $7 = \sqrt[3]{\quad}$

g. $6 = \sqrt[4]{\quad}$
h. $3 = \sqrt[5]{\quad}$

Lesson 7-5A The Root of the Problem

In this activity, we will continue to explore fractions as exponents. In particular, we want to learn to find the value of expressions like $4^{\frac{3}{2}}$ and $8^{\frac{2}{3}}$.

1. First, let's practice using some of the exponent rules we have already learned.

   a. $(2^4)^3 =$ b. $(3^5)^2 =$ c. $(5^2)^4 =$

   d. $(2^3)^4 =$ e. $(3^2)^5 =$ f. $(5^4)^2 =$

2. Now let's apply exponent rules to some simple fractional exponents.

   a. $\left(4^{\frac{1}{2}}\right)^3 = 4^{\square}$ b. $\left(4^{\frac{1}{2}}\right)^5 = 4^{\square}$ c. $\left(4^{\frac{1}{2}}\right)^7 =$

   d. $(4^3)^{\frac{1}{2}} = 4^{\square}$ e. $(4^5)^{\frac{1}{2}} =$ f. $(4^7)^{\frac{1}{2}} =$

3. Summarize in your own words what the questions above demonstrate about how exponents work.

For questions 4-7, we will think about what expressions like $4^{\frac{3}{2}}$ and $8^{\frac{2}{3}}$ mean in the internet context.

4. In the expression $4^{\frac{3}{2}}$, the base number 4 means that the amount of information available on the internet gets multiplied by ____ every _________. What does the exponent $\frac{3}{2}$ mean?

5. In a fractional exponent like $\frac{3}{2}$, the denominator indicates a ________ , so we need to find the ________ root of the base, or $\sqrt{\quad}$ = ____ . This means that $4^{\frac{1}{2}} = 2$, so we should multiply by ____ for every ______ of a year. Since we want $\frac{3}{2}$ years, or $1\frac{1}{2}$ years, let's just multiply by 2, three times: ____×____×____ = ____. Therefore, $4^{\frac{3}{2}}$ = ____ .

6. In the expression $8^{\frac{2}{3}}$, explain the meaning of the base (8) and the exponent $\left(\frac{2}{3}\right)$.

7. Find the root indicated by the denominator of the fraction: $\sqrt[3]{\quad}$ = ____ . Now multiply by this the number of times indicated by the numerator (__): ___×___ = ___. Therefore, $8^{\frac{2}{3}}$ = _____.

8. Let's put all of these ideas to work with problems involving fractional exponents.

a. $4^{\frac{1}{2}}$ = ___

b. $4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = (\quad)^3$ = ___•___•___ = ___

c. $4^{\frac{5}{2}} = \left(4^{\frac{1}{2}}\right)^5 = (\quad)^5$ = ___

d. $4^{\frac{7}{2}} = \left(4^{\frac{1}{2}}\right)^7 =$

e. $27^{\frac{1}{3}}$ = ___

f. $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2 = (\quad)^2 =$

g. $\left(27^{\frac{1}{3}}\right)^4 =$

h. $\left(27^{\frac{1}{3}}\right)^5 =$

Now we are ready to generalize how to work with fractions as exponents:

Fractional Exponents Rule #2

$$b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m \quad \text{or} \quad b^{\frac{m}{n}} = \left(b^m\right)^{\frac{1}{n}}$$

Since $b^{\frac{1}{n}} = \sqrt[n]{b}$, these can also be expressed as

$$b^{\frac{m}{n}} = (b)^m \quad \text{or} \quad b^{\frac{m}{n}} = \sqrt[n]{b^m}$$

9. Use these principles to find the values below.

a. $4^{\frac{3}{2}} =$

b. $8^{\frac{2}{3}} =$

c. $16^{\frac{3}{2}} =$

d. $27^{\frac{2}{3}} =$

e. $9^{\frac{3}{2}} =$

f. $16^{\frac{3}{4}} =$

g. $81^{\frac{3}{4}} =$

h. $32^{\frac{3}{5}} =$

i. $125^{\frac{2}{3}} =$

j. $64^{\frac{2}{3}} =$

k. $243^{\frac{2}{5}} =$

l. $64^{\frac{5}{6}} =$

m. $64^{\frac{4}{3}} =$

n. $25^{\frac{3}{2}} =$

o. $128^{\frac{6}{7}} =$

Lesson 7-5B You've Got The Power!

Time to practice what you've learned about exponents! You should be able to complete this page without using a calculator!

Simplify.

1. $25^{\frac{1}{2}} =$

2. $9^{\frac{1}{2}} =$

3. $27^{\frac{1}{3}} =$

4. $64^{\frac{1}{3}} =$

5. $16^{\frac{3}{4}} =$

6. $9^{\frac{3}{2}} =$

7. $27^{\frac{2}{3}} =$

8. $9^{\frac{5}{2}} =$

9. $8^{\frac{2}{3}} =$

10. $25^{\frac{3}{2}} =$

11. $64^{\frac{2}{3}} =$

12. $4^{\frac{5}{2}} =$

13. $81^{\frac{3}{4}} =$

14. $64^{\frac{5}{3}} =$

Use properties of exponents to write each expression with only one positive exponent. Show all your steps clearly!

15. $\dfrac{\left(2^4\right)^3 \bullet 2^{-5}}{2^6 \bullet 2^5}$

16. $\dfrac{3^{-5} \bullet \left(3^2\right)^5}{3^8 \bullet 3^{-1}}$

17. $\dfrac{4^{12} \bullet \left(4^4\right)^{-3}}{4^5 \bullet 4^{-8}}$

18. $\dfrac{\left(5^3\right)^6 \bullet \left(5^{-4}\right)^7}{\left(5^4\right)^{-2}}$

19. $\dfrac{6^{14} \bullet 6^{-6}}{(6^3)^3}$

20. $\dfrac{(7^2)^5 \bullet 7^{-6}}{\left(7^3\right)^7}$

21. $\dfrac{x^{-5} \bullet (x^4)^2}{x \bullet x^3}$

22. $\dfrac{(y^4)^2 \bullet (y^8)^{-1}}{y^3}$

23. $\dfrac{(z^{-6})^{-2} \cdot z^0}{z^5 \bullet z^3}$

24. $\dfrac{w^{21} \cdot \left(w^3\right)^{-7}}{\left(w^{18} \cdot w^2\right)^0}$

Five Exponent, Six Exponent, Seven Exponent, Or…

Simplify each expression. Use only positive exponents in your answers.

1. $a^3 \cdot a^5 =$

2. $y^{-2}y^7 =$

3. $t^{11}t^{-13} =$

4. $x^3x^{-3} =$

5. $\frac{x^5}{x^4} =$

6. $\frac{x^2}{x^2} =$

7. $\frac{y^7}{y^{-7}} =$

8. $\frac{y^{-3}}{y^{-5}} =$

9. $\frac{x^3x^4}{x^7x^{10}} =$

10. $(-3y)^3 =$

11. $(-2x)^4 =$

12. $(y^{-4})^{-3} =$

13. $\left(\frac{5}{9}\right)^2 =$

14. $(x^7)^0 =$

15. $(2y^3)(-8y^4) =$

16. $(5x^7y^4)(8x^5y^5) =$

17. $(-x^4y)^2(-xy) =$

18. $\frac{p^8q^{-5}}{p^{-5}q} =$

19. $\frac{2x^4y^7}{4x^5y} =$

20. $\left(\frac{a^{-2}b^3}{c}\right)^{-2} =$