

1-4 Reteaching

Equivalent algebraic expressions are expressions that have the same value for all values for the variable(s). For example $x + x$ and $2x$ are equivalent expressions since, regardless of what number is substituted in for x , simplifying each expression will result in the same value. Certain properties of real numbers lead to the creation of equivalent expressions.

Commutative Properties

The commutative properties of addition and multiplication state that changing the order of the addends does not change the sum and that changing the order of factors does not change the product.

Addition: $a + b = b + a$

Multiplication: $a \cdot b = b \cdot a$

To help you remember the commutative properties, you can think about the root word “commute.” To commute means to move. If you think about commuting or moving when you think about the commutative properties, you will remember that the addends or factors move or change order.

Problem

Do the following equations illustrate commutative properties?

a. $3 + 4 = 4 + 3$

b. $(5 \times 3) \times 2 = 5 \times (3 \times 2)$

c. $1 - 3 = 3 - 1$

$3 + 4$ and $4 + 3$ both simplify to 7, so the two sides of the equation in part (a) are equal. Since both sides have the same two addends but in a different order, this equation illustrates the Commutative Property of Addition.

The expression on each side of the equation in part (b) simplifies to 30. Both sides contain the same 3 factors. However, this equation does not illustrate the Commutative Property of Multiplication because the terms are in the same order on each side of the equation.

$1 - 3$ and $3 - 1$ do not have the same value, so the equation in part (c) is not true. There is not a commutative property for subtraction. Nor is there a commutative property for division.

Associative Properties

The associative properties of addition and multiplication state that changing the grouping of addends does not change the sum and that changing the grouping of factors does not change the product.

Addition: $(a + b) + c = a + (b + c)$

Multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Problem

Do the following equations illustrate associative properties?

- a. $(1 + 5) + 4 = 1 + (5 + 4)$
- b. $4 \times (2 \times 7) = 4 \times (7 \times 2)$

$(1 + 5) + 4$ and $1 + (5 + 4)$ both simplify to 10, so the two sides of the equation in part (a) are equal. Since both sides have the same addends in the same order but grouped differently, this equation illustrates the Associative Property of Addition.

The expression on each side of the equation in part (b) simplifies to 56. Both sides contain the same 3 factors. However, the same factors that were grouped together on the left side have been grouped together on the right side; only the order has changed. This equation does not illustrate the Associative Property of Multiplication.

Other properties of real numbers include:

- | | | |
|---|-------------------|-------------------|
| a. Identity property of addition: | $a + 0 = a$ | $12 + 0 = 12$ |
| b. Identity property of multiplication: | $a \cdot 1 = a$ | $32 \cdot 1 = 32$ |
| c. Zero property of multiplication: | $a \cdot 0 = 0$ | $6 \cdot 0 = 0$ |
| d. Multiplicative property of negative one: | $-1 \cdot a = -a$ | $-1 \cdot 7 = -7$ |

Exercises

What property is illustrated by each statement?

1. $(m + 7.3) + 4.1 = m + (7.3 + 4.1)$
2. $5p \cdot 1 = 5p$
3. $12x + 4y + 0 = 12x + 4y$
4. $(3r)(2s) = (2s)(3r)$
5. $17 + (-2) = (-2) + 17$
6. $-(-3) = 3$

Simplify each expression. Justify each step.

7. $(12 + 8x) + 13$
8. $(5 \cdot m) \cdot 7$
9. $(7 - 7) + 12$

(1) Associative property of addition, (2) Identity property of multiplication, (3) Identity property of addition, (4) Commutative property of multiplication, (5) Commutative property of addition, (6) Multiplication property of -1, (7) Commutative property of addition, associative property of addition, simplify, $8x + 25$, (8) Commutative property of multiplication, associative property of multiplication, simplify, $35m$, (9) Inverse property of addition, identity property of addition, 12

Lesson 1-4 Additional Practice

Simplify each expression. Justify each step.

1. $9 + 2p + 3$

2. $[4 + (-4)]y$

3. $3 + \frac{3}{18} \cdot \frac{18}{3}$

4. $8 \cdot (2y)$

Identify the property of real numbers shown in each situation.

- The cost of one item sold for \$14.50 is \$14.50.
- You can find the cost of fish by multiplying the price per pound by the amount or by multiplying the amount by the price per pound.
- To find total time spent doing homework in a week, you add the amount from each day. You find that the total is the same no matter what order you use.

Use deductive reasoning to tell whether each statement is *true* or *false*. If it is false, give a counterexample.

- For all real numbers a and b , $a - b = b - a$.
- For all real numbers x , $x \cdot 0 = 0$.